Accurate body density index BDI_n to replace flawed BMI

Robert E (Robin) Miller III ©2021^A editors: Yehia Y Mishriki MD and G H Aykroyd BSEE

Abstract: BMI is distorted for most individuals, giving the same result for any individuals of the same weight and height despite who is flabby and who is lean. A more correct measure of adiposity could improve clinicians' assessment of health risks and giving guidance for weight control, while avoiding patient anxiety. Should BMI be superseded? This paper introduces BDI_n ("BDI-sub-n"), body density index normalized within entrenched BMI categories, but optimized using a biomedical engineering model for body volume, and an online calculator.

History of BMI and its well-known issues

Two centuries since conceived and decades in clinical practice, Body Mass Index (BMI) is a distortion of individual body fat. $^{1\ 2\ 3\ 4\ 5}$ It oversimplifies body density using two measurements, weight \boldsymbol{w} and height \boldsymbol{h} . BMI = $\boldsymbol{w/h^2}$ [1]. BMI ignores girths and thus actual density that varies with body shape and leanness. 6 The National Institutes of Health (NIH) concedes "[BMI] can only be a rough guide to the degree of adiposity...People with normal BMI can have a proportion of body fat exceeding 30%." a Yet it is used despite that BMI gives the same result for any two persons the same weight & height despite who is flabby and who is lean.

Fine for a *population's average*, but not an individual. A century after its invention,⁸ BMI's formula was usurped as an *individual* metric within 11 fatness categories: "severely underweight" to "morbidly obese" and two Asian categories.⁹ Suspiciously round numbers do not differentiate male v. female, leanness (muscle), or body shape (a healthier *pear* v. a more dangerous *apple*.) b Physicians who use BMI may incorrectly assess a patient's health risks, or give poor dietary advice. And for seniors as well as young adults, being told one's BMI can trigger anxiety about self-image.

Ancient scientists knew the health implications of the ratio of fat to muscle & bone, measured by body *density*, defined as mass (weight) divided by volume, $\mathbf{w/v}$ (not $\mathbf{w/h^2}$). Archimedes' water displacement determined volume, cumbersome in practice due to fasting to purge air in the digestive tract and exhaling fully under water. In 1832 Belgian statistician Adolphe Quetelet tracked the *population* of only males' $\mathbf{w/h^2}$, where inconsistencies average out with the large numbers. In 1972, Dr. Ancel Keys (developer of Krations) coined Quetelet's Index as "BMI." ^c Actuaries used it to

Fat is less dense than muscle, bone, and water, so in water a fat person floats; a lean person sinks. If a weight increase is due to fat, BMI rises, which seems sensical, as we associate danger with rising numbers: higher blood pressure, radioactivity, pollen count. But if a weight increase is muscle, instead of falling, BMI still rises, finding lean athletes "obese." Quetelet's formula causes this reciprocity, and randomly magnifies BMI's individual error.

Is it time to replace BMI? Recognizing its male exclusivity and other shortcomings, an Oxford mathematician's "New BMI" still uses only \boldsymbol{w} an \boldsymbol{h} . And the Mayo Clinic's body volume indicator, BVI, uses too costly a scanner for routine clinical practice. This paper quantifies BMI's distortions that for most individuals, wideranging as in images below, can be between -8.4 and +11.0 points off where they ought to be, closer to BMI's own trendline in Fig.1. Then fitted to BMI's entrenched categories, the author introduces an accurate BDI_n ("BDI-sub-n"), body density index, normalized.





L: Underweight in BMI of 18.2 but healthy in BDI_n of 20.6 - Google image. R: Morbidly obese male of BMI 52.9 and waist:hips >1.0 - Wikipedia.

BMI use for individuals disproven mathematically

BMI is brilliant *for a population, but not for an individual.*¹² Averaged, its **dashed orange** trendline in Fig.1 is relevant. But should we expect any individual to conform to the average?

set progressive insurance rates, destined in time to inflate. $^{d\ 10}$ Soon BMI captured the world of medicine: the Royal College of Physicians in 1973; in 1995 by the WHO, plausibly as a % body fat, which it is not. Its categories were by observation, designating $18.5^{\circ}24.9\ (21.5\pm^{\circ}3)$ as "normal," differing from Garrow's in 1985 and others. Keys was criticized for 50yr for disregarding data that disagreed with his premise, and only men. With women's lower bone and muscle density, there has been no "normal" for women. BMI is called "bogus" by NPR, a "scam" by The NY Times.

^a Nat'l Inst Health - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6738397/

^b Higher proposed for females and with age, lower for Asians & black-Americans, who average longer legs. Category bounds were all round numbers in 1995 as first published by the WHO - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4890841/

[°] Ancel Keys-Seven Countries Study - https://pubmed.ncbi.nlm.nih.gov/31338844/; https://pubmed.ncbi.nlm.nih.gov/31338844/; https://www.scienceofeds.org/wp-content/uploads/pekar-ims-bmi.pdf (only males).

d 1943 – MetLife-Weight for Height. http://www.assessmentpsychology.com/metlife.htm; updated 1972 as BMI - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4904092/; http://www.assessmentpsychology.com/metlife.htm misusing Quetelet's w/h².

^e N Trefethen, Prof Numerical Analysis, Univ. of Oxford, "New BMI"=1.3*w/h^2.5; C Scamahorn "BFATI" [Old BDI"] 2006=W(w+b)/h^3 kg*mm/mm^3 [~kg/mm^2??]

f A 19.4-point spread; later the same point spread for BMI cf. BDIn of -6 to +13.4.

BMI describes as "normal" a healthy category mean of 21.5, e.g. an 1830's Belgian of 57.73kg (127lb) and 1.638m (64.5in).

BMI = w/h² = 57.73 / 1.638² **= 21.52** ostensibly
$$kg/m^2$$
 [2]

or in inch-pound units common in North America and elsewhere

BMI = w/h² = 703 x 127lb / 64.5
$$in^2$$
 = **21.46** " kg/m^2 " [3]

In either, BMI is kg/m^2 , not density. Bodyfat equates to density, mass per unit volume, $\mathbf{\varrho} = \mathbf{m}/V$. In earth's gravity $\mathbf{\varrho} = \mathbf{w}/V$. Viewing \mathbf{V} as a cube $\approx hhh$, then $\mathbf{\varrho} \approx w/hhh$. But BMI's calculation is $\approx w/hh$. Shy an h in its denominator effects an extra h in its numerator, as in substituting $\mathbf{\varrho} \mathbf{V}$ for $\mathbf{w} \approx \mathbf{\varrho} hhh$, then BMI $\approx w/hh$ is $\approx \mathbf{\varrho} hhh/hh = \mathbf{\varrho} h$. With no shape factors but h^2 , BMI cannot present body leanness, miscalculating density by not using \mathbf{V} and producing a rogue h. g 13

Therefore BMI as a physical density is meaningless, and in clinical examinations it is typically expressed as dimensionless. Clinicians do not calculate BMI, but look it up in a printed table or online *widget*, and report only a scalar number – an "index." This paper still uses the term index, but defines a meaningful density-based indicator of adiposity termed BDI_n to supersede BMI.^h

Fig.1 plots the small "population" in Table 1 spanning individual BMI or BDI_n ranging between 14 and 53 v. body densities between 0.737~1.130 g/cm^3 . Shading delineates conventional BMI subcategories, "severe underweight" <16.5 to "morbidly obese" >40. (A tightly grouped centered trendline implies a healthy population with fewer people morbidly obese or dangerously underweight.)

BMI is immutable careening about its own dashed orange trendline, because its calculation uses only **w** and **h**², ignoring individual shape and leanness. This works statistically only if averaging a large number of people. However taken by individual body density BD, then BMI (solid orange line) errs between –8.4 and +11.0 points (vertically) from to its own trendline, swinging between –29% and +39%. Were it accurate for individuals, BMI's datapoints would be linear, falling on its dashed trendline. This paper's BDI_n in blue is normalized to parallel BMI's trendline, and to contain its normal 21.46. BDI_n datapoints directly below or above corresponding BMI points differ by between –6 and +13.4 points, implying individual corrections between +17% and –90%. Derived from a truer body density BD, proposed BDI_n for the same individuals are inherently linear, without miscalculation distortion.

Do not expect BDI_n to fall on BMI's averaged trendline, which is altered with each added dataset. Whether or not BMI's arbitrary categories are to be retained or improved on an objective basis, individual BDI_n's plot accurately within them. The key is a more accurate body density $BD=w/V_{body}$ from a truer body volume.

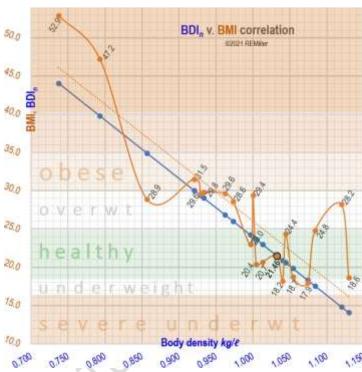


Fig.1 - Plotted v. body density for Table 1's "population," **BMI** by individual is erratic, often far from its own trendline - - -. (Other replacements "New BMI" and "BFATI" are in Appx C.) *Normalized* to familiar BMI categories, **BDI**_n is linear (undistorted), and therefore is the truest *individual* index of fatness.

Undistorted BDIn from an accurate body volume

An accurate index of body fat derives from density, $\varrho=m/V$. In space, mass m just sits there, but on earth, gravity converts it to a force of weight \mathbf{w} . Then $\mathbf{w/V}$ gives body density \mathbf{BD} for any male, female, lean\athletic, or aged body shape. Its accuracy depends on a \mathbf{V}_{body} that gives a better result than BMI's h^2 – far better. For any shape, BDI_n provides a true, individual index of body health.

Body volume V_{body} is measured not by dunking or an expensive scanner, but by a biomedical engineering model. In Fig.2 L, this paper's model calculates V_{body} from a minimum number of five (5) size measures: the usual overall height h, plus an additional four: sitting height h_{sit} , waist depth d, and perimeters waist p_w and hips p_h that with practice take only about a minute more to measure. Greater precision calls for measuring to the nearest $\frac{1}{2}$ in or $\frac{1}{2}$ cm.

Simply put, the model is an elliptic cylinder for the trunk atop two truncated cones for legs. $V_{body} = V_{trunk} + V_{legs(2)}$ [4]. Fig.2L shows the cylindrical trunk, integrating the head, neck, & arms by clasping hands behind head and tucking elbows in and up even with the crown. Arm volume is assumed proportional to the trunk. As brain density is higher than average BD, the head is in effect a larger volume, dealt with after a 1st approximation.

^g A Journal of the American Heart Association 2018 study finds girth measures a better indicator of heart attack risk than BMI. Table1 uses 0.78 ratio of waist:hips.

^h BDI_n when normalized to BMI, otherwise BDI_x is extensible for any calibration.

ⁱ Table 1 does not represent the US pop., which per CDC is on average borderline obese, but shows how BMI wrongly categorizes both the unhealthy and healthy.

j Density of water by definition equals 1.000 g/cm³ (1.000 kg/ℓ), or ~0.036 lb/in³.

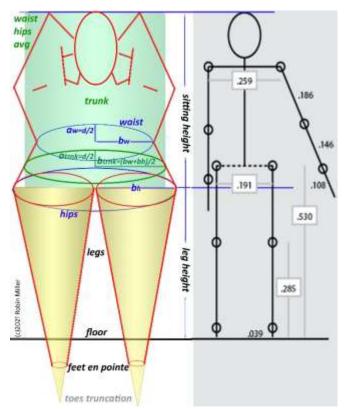


Fig.2 L – BDI $_n$ is based on a sufficiently accurate model of body volume, an elliptical cylinder + two cones, from five measurements. R - Mean unit ratios of anthropometrics of Drillis, Contini segmented by joints. ^k ¹⁵

NB: The math-averse may wish to skip the rest of this section.

An ellipse (the special case being a circle of a rotund person) has semi-minor & semi-major axes a & b and area $A = \pi ab$. Then

 $V_{trunk1st} \approx \text{Area}_{\text{base}} * h_{\text{sit}} \approx \pi a_{trnk} b_{trnk} h_{sit} \approx \pi (d/2) b_{trnk} h_{sit} m^3$ [5] where $a_{trnk} = d/2$, the measured trunk depth, h_{sit} is sitting height, and b_{trunk} is calculated next from waist & hips perimeters $p_w \& p_h$.

NASA published data for 40yr-old Americans in 2000.^m The 50th percentile ratios of hip breadth, $2*b_h$ in Fig.2L, to generalized bust depth d is 0.6510 for men or 0.6083 for women, 14 each within $^{\sim}4\%$ of their average of **0.630**. Assuming the hips in typical cross-section is determined by hipbone geometry, then hips is an ellipse of this ratio, $a_h:b_h$ (a_h not shown), dimensioned by hips perimeter p_h . We find b_h from p_h . Then a_h is substituted by 0.630 b_h .

For any ellipse the relationship between a & b and perimeter p involves integral calculus. Happily an estimate of p within 2% is given by the root mean square (RMS) $p=2\pi V((a^2+b^2)/2))$, then ⁿ

$$b_h^2 = p_h^2 / 2\pi^2 - 0.63^2 b_h^2$$
, $1.3969 b_h^2 = p_h^2 / 2\pi^2$, $b_h = 0.1904 p_h$ m [6].

Unlike the hips ellipse, the waist of perimeter p_w is an ellipse where $a_w = a_{trnk} = d/2$ from the measure of trunk depth, from a flat stomach to an obese circle, or greater. The waist semi-major axis

$$b_w \approx V(p_w^2/2\pi^2 - a_w^2) \approx V(p_w^2/2\pi^2 - (d/2)^2)$$
 m [7].

Fig.2L shows a trunk volume of h_{sit} times the area of an ellipse that is imagined averaging the waist and hips (either one larger). Substituting in [5] a trunk half-breadth average $b_{trnk} = (b_h + b_w)/2$

$$V_{trunk1st} \approx \pi(d/2)b_{trnk}h_{sit} \approx h_{sit}\pi(d/2)(b_h+b_w)/2$$

 $\approx \frac{4}{\Delta}h_{sit}\pi d(0.1904p_h+v(p_w^2/2\pi^2-(d/2)^2)) m^3$ [8].°

This 1^{st} approximation of trunk volume will be completed in the *Errors...uncertainties* section to correct by a factor Δ for voids not filled by other trunk features or the head's equivalent volume.

With better nutrition and less disease, we've evolved longer-legged than da Vinci's *Vitruvian Man*. Leg height is *not* defined by the joints in Fig.2R, but standing height h minus h_{sit} that extends from below the hip joints. A first approximation of a leg volume is $\frac{1}{2}L_{leg}$ times the cone base area of radius $b_h/2$, so $r_{leg}=0.0952p_h$.

$$V_{leg1st} \approx \frac{1}{3}A^*L_{leg} \approx \frac{1}{3}\pi r^2(h-h_{sit}) \approx \frac{1}{3}\pi (0.0952p_h)^2(h-h_{sit}) m^3$$
 [9].

Then the 1st approximation of the volume of two legs simplifies to

$$V_{legs(2)1st} \approx \frac{2}{3}\pi 0.00906 p_h^2 (h - h_{sit}) \approx 0.018981 p_h^2 (h - h_{sit}) m^3 [10].$$

Refining the model relates to the feet. In Fig.2L the dancer is *en pointe*, but un-squooshed, as in zero gravity. Foot length averages $\sim 0.1515 \, h$, q 15 or $\sim 0.3030 \, L_{leg}$ for $L_{leg} \sim h/2$. By Fig.2R, the ankle joint is above the floor at $0.039 \, h$, or $\sim 0.078 \, L_{leg}$. L_{leg} elongates toes to virtually below the floor by $0.3030 - 0.078 \, \approx 0.225$. So the 1^{st} of 3 adjustment coefficients applied to L_{leg} is $1.000 + 0.225 \, \approx 1.225 \, L_{leg}$.

By observation the dancers' toes do not come to a point, but truncate the leg cone visualized in a circle of diameter $^{\sim}b/5$ after extending L_{leg} further by $^{\sim}1/5$. The truncation area is a squared fraction of the base cone, or $1^2/5^2 = 1/25$; the truncation volume a cubed ratio $^{\sim}1^3/5^3 = 1/125$ or a factor of $^{\sim}0.992$. The three leg cone refinements combine to [1.225*1.20*0.992] = 1.458 as in

$$V_{legs(2)} \approx 0.01898 p_h^{2*} L_{leg}^{*} 1.458 \approx 0.0277 p_h^{2} (h - h_{sit}) m^{3} [11].$$

As said, the most critical in determining body density is volume. Perfection is not expected, only that one's body density be weight divided by a volume more accurate than BMI's using h^2 . From [4] with **inputs in bold**, and Δ is a model correction factor described later, the total $V_{body} = V_{trunk} + V_{legs(2)}$, is combined in

 V_{body} ≈Δh_{sit}πd(0.1904p_h+ $\sqrt{(p_w^2/(2\pi^2)-(d/2)^2)})/4+0.0277p_h^2(h-h_{sit})$ m³ [12].

^k Drillis & Contini 1966 Body segment parameters #1166.03, New York University

When the ellipse is a circle, a = b = r, the radius, and area is πr^2 .

m https://msis.jsc.nasa.gov/sections/section03.htm – The US mean age is 38yr. Note NASA's typo "Japanese female" titling the chart for American female. Other anthropometric data at - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2872302/

 $^{^{}n}$ p=2π√((a²+b²)/2)), p²=2π²(a²+b²) =2π²a²+2π²b², then b²=p²/2π²-a².

[°] V_{trnk} ≈ $h_{sit}\pi(d/2)^*(b_h+b_w)/2$ ≈ $\frac{1}{4}h_{sit}\pi d^*(b_h+b_w)$.

^p Sitting height h_{sit} measured per Appx A; leg length (subischial) $L_{leg} = h - h_{sit}$.

^q Univ RI Electrical, Computer & Biomed Eng; K T Davis, TX Tech Univ 1990 https://ttu-ir.tdl.org/bitstream/handle/2346/8468/31295005963201.pdf?sequence=1&isAllowed=y

From 1970 to 2019, U.S. mean age rose from 28yr to 38yr. Per *Appx B*, Americans today average 50lb heavier and 8 points higher in BMI. Yet we use 1995 BMI fatness categories, with "normal" equivalent to a 1830s Belgian 127lb and 5ft 4½in. While 2in taller today one can have a leaner *look*, BMI and its categories do not register fat v. muscle v. bone, gender, age, or waist to hips ratio.

Humans only $^{1}/_{13}$ cubic meters in mean volume suggests that a body density **BD** in *metric tons per cubic meter* is hardly intuitive. A more understandable body density BD is in kg/liter ℓ (= g/cm^{3}):

BD \approx Cw/ Δ V_{body} kg/ ℓ (inputs metric or converted Imperial) [13]

where units factor **C**= $1000cm^3/\ell$ for metric or 27.68 $in^3lb/kg\ell$ (61 $in^3/\ell \div 2.2046lb/kg$) for Imperial inputs. Then density is

BD
$$\approx$$
 Cw/($\Delta h_{sit}\pi d(0.1904p_h + \sqrt{(p_w^2/(2\pi^2) - (d/2)^2)})/4 + 0.0277p_h^2(h - h_{sit}))$ kg/ℓ [14]

where w is weight in kg or lb, and p's and h's are in cm or inches. From Imperial inputs in lb & in, BD_{imp} is converted to metric kg/ℓ .

For the "normal" body of **w** 127lb (57.73kg), **h** 64.5in (163.8cm), extrapolating **h**_{sit} 34in (86.36cm), **waist** 28.0in (71.12cm), **d** 8.5in (21.59cm), **hips** 35.0in (88.90cm), modelled body density BD is

$$\approx 1000*57.73/((\Delta*84.46*3.1416*21.6*(0.1904*91.44+(73.66^2/(2*3.1416^2)-(21.6/2)^2)^0.5)/4+0.0277*91.44^2*(163.8-84.46))) $\approx 0.985kg/\ell$ [15] ^r$$

close to pure water, $1.00 \text{kg/}\ell$ by definition. Fat is $\sim 0.9 \text{kg/}\ell$; muscle ~ 1.1 ; bone up to 1.38. Humans are $\sim 15\%$ bone; $\sim 85\%$ fat, muscle & watery fluids. 17 So BD falls below the weighted inequality

BD <
$$85\%*1.1+15\%*1.38$$
 BD < 1.142 kg/ ℓ [16]

bounded by 0% muscle (impossible so not shown) and 0% fat (2% is possible). An expected albeit narrow range of body density BD will be well below 1.142. BMI's formula only poses for "BD," but its calculation inverts and amplifies its form of "BD" by ~76 times.

Now to transform BD to an equally sensitive index BDI_n that is as familiar to health providers & patients as BMI, but undistorted.

After 50yr universal use, BMI is entrenched in the healthcare community, along with its round-number categories. It would be ergonomic if actual body density BD were comparable to BMI's familiar scale. Termed body density *index* BDI_n ("BDI-sub-n"), it is *normalized* to reflect BMI's trend line equation y = Mx + N. Then with Fig.1 slope M=76.42, the BD inequality limit of 1.142, and the BMI_{1830norm} of 21.5=M*(1.142-BD)+N, we solve for N=13.12, and

BDI_n = **76.42*** (**1.142–BD**) + **13.12** (
$$kg/\ell$$
 normalized index) [17].

The complete expression for "Body Density Index, normalized":

BDI_n
$$\approx$$
 N + M * (1.142 – Cw/($\Delta h_{sit}\pi d^*$ (0.1904 p_h + $V(p_w^2/(2\pi^2)$ -($d/2$)²)^{5.5})/4 + 0.0277 p_h^2 ($h - h_{sit}$)) (kg/ ℓ normalized index) [18].

Constants M & N slope BDI_n to any categories, C is 1000 for metric or 27.68 for Imperial inputs, Δ is a model correction. Variables W is weight in K or K or K is are waist K hips perimeters, K is trunk depth, K are standing K sitting heights. BDI_n's six body measurements are in **bold italic**. Its four measurements in addition to W K or BMI take about a minute extra to make.

Applying an accurate body density index BDIn, normalized to BMI

Table 1 calculates BDI_n v. BMI for 20 input datasets, graphed in Fig.1, including 2021 averages of the United States Centers for Disease Control & Prevention (CDC). Each row is an individual. For illustration, unavailable measurement data are extrapolated in round numbers. Rows are sorted by body density BD to plot in Fig.1. Center-right are four indices: proposed BDI_n, BMI, "New BMI" that still uses only **w** & **h**, and so-called "BFATI" roughly per its formula (with sizes in *mm* to force its results into the ballpark).

Table 1 "Averages" shows **BD** 0.985, but **BMI** 5.7 points higher than its own "norm." And individual rows are between –8.4 and +11.0 points off BMI's own trendline. Healthy individuals whose BMI indicate underweight or obese are radically higher or lower in BDI_n ("differ" and Fig.1"). #1 (on p1) through 4 are at high risk.

#3 measures tall, but his caption notes a trunk maximum of 47.5*in* above the waist and a minimum of 44.5*in*. Although not obese per a BMI of 28.9, he is at risk 6 points higher in BDI_n.

#5 & 8 is the author at 215/b before and 208/b mid-diet, a 1-point drop in BMI, but 3+ points lower BDI_n with an inch reduced waist, depth, & hips. Most loss is in his trunk, equivalent to 3.2 liters of H_2O , equating to the 7lb weight lost. (His goal is <200.)

#6 & 10 are not individuals, but CDC-average American woman or man included for reference. Females remain borderline obese in BMI and BDI₀ while males in BDI₀ reclassify to just overweight.

#12 has a DEXA-scanned bone deficiency, making her total BD high for her weight & petite figure, and caught by her high BDI_n.

#14 is the female on p1 who in BMI is underweight, but healthy in BDI_n . 15 is near overweight by BMI, but normal in BDI_n .

Runway model #17 is expected underweight in BMI & BDI_n. $^{\text{v}}$ 20 is deemed healthy in BMI, but is dangerously underweight in BDI_n.

^r The consensus online of average human body density is 0.985kg/liter.

s ~1.38 Zioupos et al Bone Density https://doi.org/10.1016/j.jbiomech.2008.03.025

^t Anywhere a BMI datum is on its trendline is valid to find N=BMI_n-M(1.142-BD_n).

 $^{^{\}rm u}$ Fig.1 plots indices against modeled BD that renders BDI $_{\it n}$ linear; a perfected $V_{\rm body}$ may require slightly rescaling BDI $_{\it n}$, but will not render others less erratic.

[∨] Scaled to 5'9" 110*lbs*, Barbie's BMI is 16; adding waist 18*in* & hips 33, BDI_n is 9.

BDI v. BMI (Imperial) SEMBer210811a			M _{slope} = 76.420 N _{core}			13.12	Δ= 0.9250 0.712-1.442		0.712-1.142	y=Mx+N-BM trendline		1.3wh25 W/w+bjh3	BMI - BDI		BM y TrendLine		ideal<0.80	
Sbj Catgry	w ib	∆CDC	h _{mand} in	∆CDC	h _{ab} /n	waist in	dep in	hips in	BD .	BDI,	BMI .	"New"BMI	BFATIm	differ .	differ%	BMI	BMI _{VTL}	p _w : p _h
1 & Vhvy	401.0	230.2	73.00	4.0	40.00	65.00	20.0	55.00	0.737	44.1	52.9	50.5	79.8	8.8	20%	6.5	12%	1.18
2 ⊊Vhvy	250.0	79.2	61.00	-2.5	34.00	55,00	13.0	53.00	0.793	39.8	47.2	49.3	76.7	7.5	19%	5.1	11%	1.04
3 ♂Sr hvy¹	228.0	57.2	74.50	5.5	35.50	46.00	13.0	42.50	0.857	34.9	28.9	27.3	31.5	-6.0	-17%	-8.4	-29%	1.08
4 ♀Sr hvy	175.0	4.2	62.50	-1.0	32.50	40.00	10.5	42.00	0.921	30.0	31.5	32.5	37.9	1.5	5%	-0.9	-3%	0.95
5 d'Sr Lg3	215.0	44.2	71.50	2.5	35.50	41.00	12.0	42.00	0.929	29.4	29.6	28.5	31.5	0.2	1%	-2.1	-7%	0.96
6 ♀CDC2021	170.8	0.0	63.50	0.0	33.40	37.95	9.5	43.00	0.933	29.1	29.8	30.5	34.8	0.7	2%	-1.6	-5%	0.88
7 ♀&∂CDC	185.3	14.5	66.30	0.0	35.24	39.60	9.6	42.00	0.963	26.8	29.6	29.7	33.5	2.8	10%	0.5	2%	0.94
8 d'Sr Lg3diet	208.0	37.2	71.50	2.5	35.50	40.00	11.0	41.00	0.974	26.0	28.6	27.6	29.7	2.6	10%	0.3	1%	0.98
9 ♀fit 40\50	138.0	-32.8	65.00	1.5	35.00	29.00	9.0	37.00	0.997	24.2	23.0	23.2	21.4	-1.2	-5%	-3.6	-16%	0.78
10 3 CDC2021	199.8	29.0	69.10	0.1	37.00	40.08	9.8	41.00	1.000	24.0	29.4	28.9	31.7	5.5	23%	3.1	11%	0.98
11 ♂Sr Md	138.0	-32.8	69,00	0.0	35.75	32.00	8.0	35.50	1.005	23.6	20.4	20.0	18.3	-3.2	-14%	-5.6	-27%	0.90
12 ♀Sr xSm²	104.0	-66.8	59,50	4.0	28.00	28.50	6.9	36.00	1.013	23.0	20.7	21.8	20.5	-2.3	-10%	-4.7	-23%	0.79
13 1830"norm"	127.0	-43.8	64.50	-1.8	34.00	28.00	8.5	35.00	1.033	21.48	21.46	21.80	19.24	0.0	0%	-2.4	-11%	0.80
14 ⊋yng adult ⁴	123.0	-47.8	69.00	5.5	36.00	24.00	6.9	37.00	1.040	20.9	18.2	17.8	14.7	-2.7	-13%	-5.1	-28%	0.65
15 d fit 40\50	165.0	-5.8	69.00	5.5	36.00	34.50	8.5	38.00	1.044	20.6	24.4	23.9	23.5	3.8	18%	1.5	6%	0.91
16 d'yng adult	142.0	-28.8	73.00	4.0	35.00	28.00	8.5	35.00	1.054	19.8	18.7	17.9	14.8	-1.1	-6%	-3.4	-18%	0.80
17 ⊋tall model	125.0	-45.8	70.00	6.5	35.00	25.00	7.9	34.00	1.074	18.3	17.9	17.5	13.9	-0.4	-2%	-2.7	-15%	0.74
18 ♀fitns mod	140.0	-30.8	63.00	-0.5	33.00	27.00	8.6	38.00	1.084	17.5	24.8	25.5	23.5	7.3	41%	4.9	20%	0.71
19 ditns mod	180.0	9.2	67.00	3.5	33.00	34.00	9.8	38.00	1.120	14.8	28.2	28.1	27.8	13.4	90%	11.0	39%	0.89
20 ♀\∂*anorex	122.5	-48.3	68.00	6.5	34.00	23.00	7.9	34.00	1.130	14.1	18.6	18.4	14.3	4.6	32%	2.2	12%	0.68
Inputs measured or extrapolated per available data for illustration.					×	trpNASA	Avgslvar:	0.985	25.1	27.2	27.0	30.0	-6.0	13.4	-8.4	11.0	0.88	

Table 1 - Calculated **BD** and **BDI**_n v. **BMI** for 20 body datasets in yellow columns (Imperial units), sorted by increasing body density BD. Illustrative data is measured in black or extrapolated in green. Comparisons with CDC averages are in gray; Waist:hips ratios at far right. "New BMI"=1.3w/h $^{2.5}$ is still based only on w & h. The BMI-BDI_n "differ" and "BMI_{VTL}" columns and, across the bottom, their averages or descended min\maxima compare performance of BMI v. BDI_n showing (in Fig.1). BDI_n is more accurate by individual.

Athletes often score overly high in BMI: Female fitness model #18 is nearly over-weight in BMI, but underweight in BDI_n. #19 tends toward obese in BMI, but in BDI_n is severely underweight.

BDI_n's trendline is made to parallel BMI's and contain its normal 21.5, but individuals will fall on BDI_n's trendline, not BMI's. Table 1 & Fig.1 show BMI's radical swings among individuals, as they vary from their own trendline between -29 and +39% ("BMI_{vTL%}"). For nearly all individuals, BMI "differs" from BDI_n between -6 lower to +13.4 points higher. Not perfect, BDI_n need only be far better by individual than BMI to cover its "cost" in practice.^w

Examples calculating BDIn and comparing to BMI

"Quetelet's Index" (BMI) intended to track a population trend, not serve as a standard for any individual. Yet those individuals' erratic BMI's average to the trendline in Fig.1. The orange data points whiplash with respect to this line, while BDI_n 's in blue are linear, derived directly from individually accurate body volumes and densities, so are non-distorting by body shape, and consider sex, age, race, and leanness. Next, $Ex1^{\sim}3$ show how BDI_n is easily calculated using the online copy&search expression in Appx A...

13.12+76.42*(1.142-27.68***127**/((0.925***29***3.14***8.0***((0.1904***34**+(**30**^2/(2*3.14^2)-(**8.0**/2)^2)^0.5))/4)+0.0277***34**^2*(**64.5-29**))) [copy&search, per Appx.A]

BDI_n \approx **14.96** *cf. BMI* **21.5** ["normal" in BMI but anorexic in BDI_n]

Ex2: Find BDI: **w**57.6kg, **h**163.8cm, **h**_{sit}86cm, **p**_w86cm, **d**25cm, **p**_h86cm:

13.12+76.42*(1.142-1000*57.6/((0.925*86*3.14*25*((0.1904*86+(86^2/(2*3.14^2)-(25/2)^2)^0.5))/4)+0.0244*86^2*(163.8-86))) [copy&search, per Appx.A]

BDI_n \approx **30.18** *cf. BMI* **21.5** ["normal" in BMI but obese in BDI_n]

Note: Ex1&2 are identical in weight & height for identical BMI, but different in BMI-ignored waist, d, & hips that BDI_n includes.

Ex3: Try it. Put your own measurements in either formula above, or in Appx A, to compare your BDI_n and BMI. Any surprises?

Error compensation, propagation of uncertainty, other variables

Table 1's column "BMI $_{\rm vTL}$ " has erratic individual errors due to BMI's calculation. For a different reason, BDI $_{\rm n}$ could err by as much as 0.15ℓ (~4.5oz of fat) due to sloppy measurements. Off by

¹ high waist 47.5*in*; low 44.5*in*

² low bone density DEXA scan

³ before & after diet losing 7lb

⁴ does not appear anorexic

⁵ likely far from overweight

Ex1: In the appropriate measurement units, calculate using Appendix A BDI_n for: weight 127lb, h 5ft4½in, h_{sit} 29in, waist 30in, d 8.0in, hips 34in:

w Jo Craven McGinty - https://www.wsj.com/articles/youre-overweight-or-are-you-11628847001?st=16n04rqqtkgnqqo&reflink=article_copyURL_share

 $\frac{1}{2}$ inch (~1*cm*) in any three of the five size measurements could cause such an error. BDI₀ results are only as good as its inputs.

Math above is carried to at least three significant digits after rounding, four if a "1" is leading. Some expressions use " \approx " for "approximately equal." With 20 subjects, Table 1 and Fig.1 are sensitive to each added dataset. Statistical analysis of variance (ANOVA) of a range of subjects (\geq 601) would settle variables **M**, **N**, & Δ as constants. Clinical trials could provide that data.

BDI_n's volume model relies on assumptions, deemed small cf. overall error. E.g. average head density is ~1.1 g/cm^3 , its volume equivalent to ~10% larger.* ¹⁸ Bodily "features" beyond the trunk cylinder in Fig.2L may not entirely fill voids above the shoulders. To compensate, $\Delta \approx 0.925$ brings the average BD to the accepted 0.985 kg/ℓ for an ~7.5% smaller V_{trunk} to complete expression [18] for BDI_n. Procedures in Appx A assure results superior to BMI. ^y

Discussion & further work

One need only visit an American mall to observe all sizes and shapes of women's and men's bodies: wannabe runway models and long-torsoed-short-legged individuals who defy $h_{sit} \approx L_{leg}$. With waist:hips ratios higher\lower than a desirable 0.78. BMI ignores salient differences to mislead about individual adiposity.

Humans have evolved older, taller, & fatter since the inception of BMI. In 1830, one "normal 21.5" and with body density >1.0 is petite & lean compared to today's heftier American, whose mean ~0.985kg/l shows that fat has overtaken muscle. Fat weighs less than muscle: a fat person weighs less per liter than a lean. Yet as the proportion of muscle goes up, BMI goes up! Most individuals in Table 1 are mis-categorized in BMI, five as borderline obese who are not, two as healthy who are severely underweight.

In Appx C, other indices are plotted against the same modeled body volume and density BD that renders BDI_n linear. V_{body} from more datasets may slightly re-slope BDI_n's trendline, but will not render the other indices any less erratic. Greater precision than this paper's biomedical engineering model may be unnecessary, and entails more size measurements, e.g. caliper measures of hip depth or leg breadth, that would be too awkward for both patient and clinician's tech (albeit less awkward for a pathologist!).

Some interviewees report being told their BMI was emotionally devastating to their self-image. A truer, less distorted BDI_n could ameliorate many of these reactions, and allow patients to focus undistracted on heeding physicians' advice about their health.

Several sets of arbitrary categories of adiposity for BMI have been established, the latest in 1995 adopted by global health authorities: nine categories from "severely underweight" to three classes of obesity with bounds of 30, 35, & 40. These suspiciously round number sets (except two *band-aid* categories for Asians) have not presented underlying adiposity consistently. BDI_n's greater precision warrants more nuanced categories by gender, race, and age group (young adults, middle-aged, seniors), easily implemented by check boxes in a calculator app. An app would be agnostic to round numbers, even hiding them from comfort-seeking humans. Should a consensus arise for more objectively precise boundaries, the extensible form BDI_x before normalizing implies only rescaling M & N. Changes in categories do not diminish BDI_x or BDI_n as the successor metric of adiposity.

Either additional categories or a rescaled BDI_x could consider those who have lost all or parts of limbs. Or school-age children, or those short of stature from illness, dwarfism, or Down's Syndrome for whom (arm-span + L_{leg})/h = about 2.7 h_{sit} , or about 10% lower than that ratio for adults of normal stature.^z

Conclusions

This paper introduces BDI_n, body density index normalized, an individual and accurate index of adiposity to replace BMI. Body Mass Index (BMI) only works well averaging many individuals, but it does not determine the bodyfat of any one individual. Imposing an arithmetic average on a person regardless of fat v. lean density is bogus. BMI was hijacked for quantifying an individual's obesity to set life insurance rates! And uncritically adopted by the global healthcare community, who read a patient's obesity from a chart.

Typically recorded as three significant digits belies BMI's actual precision. BMI = w/h^2 , weight divided by 2^{nd} order height squared. It is not a true body density, the proper measure of leanness that is weight divided by an actual 3^{rd} order volume. Individual BMI's switch back & forth erratically between -8.4 to +11.0 points away from its own trendline, where they ought to be, as shown in Fig.1. Intended as a statistical average for a large group (Belgium), it is wildly inaccurate in characterizing an individuals' health, because its two body measurements are insufficient, ignoring shape, age gender, and race. Ex.1 & 2 show any two individuals identical in weight & height have the same BMI despite who's fat and who's lean. BMI rightly averages many to its trendline, but at any individual datapoint its formula is often a gross miscalculation.

Both Keys' BMI and Quetelets' Index before it exclude women, and the wide range of body shapes of many individuals and races. With women's lower bone & muscle density, there has been no "normal" for women, imposing on them the arbitrary $21.5 \pm^{\sim} 3.0$ for men. Another researcher writes: "Use of BMI as a measure

x Barber, Brockway, Higgins 1970-"The density of tissues in and about the head."

y Table 1's column "pw:ph" uses BDI_n data to give individual waist-to-hips ratios.

^{*}https://www.researchgate.net/publication/5462786 Auxology Is a Valuable Instrument_for_the_Clinical_Diagnosis_of_SHOX_Haploinsufficiency_in_School-Age_Children_with_Unexplained_Short_Stature/download

of adiposity and predictor of health risks [cardiovascular disease, stroke, type 2 diabetes, cancer] requires an understanding of its limitations for an individual. BMI accounts for appreciable variance (60–70%) in measured fatness in groups of adults...it is an unreliable indicator of the body composition of an individual."¹⁹ BDI_n by design is inclusive of all individuals.

BDI_n is based on an *individually approximated volume* from a biomedical engineering model of body shape using minimal (four) additional measurements that are convenient for both clinician's tech and patient. The model need not be perfect, only better than BMI's using only one measurement (height squared) for volume. From an accurate volume is found body density BD, and from BD, a scalable index BDI_x. For mainstreaming, BDI_n then is normalized to BMI's familiar categories and sensitivity (76 times a change in BD). BDI_n as calculated in *Appendix A* translates as better clinical health assessment and advice re weight-related issues, and with fewer unintended mental health consequences for patients distracted from that advice by body image issues.

BDI_n is an adiposity index in kg/ℓ normalized to BMI categories. For most individuals, BMI distorts body fatness; BDI_n is linear, accurately based on body density for any individual body shape and leanness, as Fig.1 & Table 1 reveal. Tested alongside BMI, BDI_n would confirm BMI's error of between -6 to 13.4 points. Its shortcomings quantify BMI as unsuitable to continue in clinical use as a measure of adiposity, and BDI_n as a suitable successor.

Appendix A - BDI_n measurement methods and calculation

BDI_n results are only as good as measurements to the nearest ¼in or ½cm and ½lb or ¼kg. Do not use clothing sizes that typically are smaller than real. Subject to relax (no sucking-in). Six BDI_n measurements are:

- w weight (same as BMI) to the nearest ±1/2lb or 1/4kg;
- h standing height (same as BMI) to the nearest ¼in or ½cm;
- p_w waist perimeter (around small of the back, standing) $\pm \frac{1}{4}$ in or $\frac{1}{2}$ cm;
- p_h hips perimeter (around the rump's widest, standing) $\pm \frac{1}{4}$ in or $\frac{1}{2}$ cm;
- d depth of trunk (from small of the back, standing) ±1/4in or 1/2cm; aa
- h_{sit} sitting height ("crown to rump" seated on a firm surface, with feet suspended for no pressure) to the nearest $\pm 1/4$ in or 1/2cm.

More than two inputs preclude a simple printed chart for BDI_n. Pending an on-line calculator [planned], copy & paste the 2-line BDI_n expression in violet below in an online search field, e.g. Google, or spreadsheet cell, substitute (select precisely & overtype, no spaces) the units constant C and all *measured inputs in bold italics* in the same units as C (1,000 for metric or 27.68 for Imperial units), and press Enter.^{bb} The result is BDI_n...

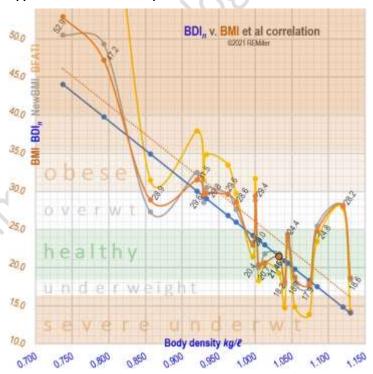
= $13.12 + 76.4*(1.142 - C*w/((0.925*h_{sit}*3.14*d*((0.1904*p_h+(p_w^2/(2*3.14^2)-(d/2)^2)^0.5))/4) + 0.0277*p_h^2*(h-h_{sit})))$

Appendix B – CDC 2021 average h, w, & p_w of Americans \geq 20yr cc

Height h Me	en: 69.0	Women: 63.5	Overall: 66.25 in
Weight w	199.8	170.8	185.3 <i>lb</i>
*BMI=29.7, BD	Nn=22.8	*BMI=29.8, BDIn=25.3	BMI=29.5, BDIn=25.4
Waist p_w	40.5	38.7	39.6 in

*Prior 2015~18 ranges: US adult males BMI 24.2~30.1; females BMI 24.2~32.9 although by 2021 males on average became 4% & females 6% heavier, and up a full point in BMI - https://www.cdc.gov/nchs/data/series/sr_03/sr03-046-508.pdf

Appendix C - other BMI replacements: "New BMI" and "BFATI"



Compared to this paper's BDI_n, "New BMI" and "BFATI" are as erratic by individual as BMI as they are based on similar density miscalculating formulae.

The author, editors, & reviewers

Internationally recognized engineer, graduate professor, pianist\ orchestrator, and Peabody winning filmmaker **Robin Miller** has communicated health issues from cardiac catheterization methods to physical & mental disabilities. dd He's presented psycho-acoustic research and his Patented full-sphere 3D sound reproduction to symposia in the US, Canada, Germany, Austria, The Netherlands, and Italy. A protégé of an inventor of the ENIAC computer, his company Filmaker Technology works in systems design, integration, testing, & publications at www.filmaker.com. With this paper he solves a biomedical engineering problem with an applied scientist's sense of what can be economically modeled and implemented. A

Aggregating data will calculate new men's and women's "normals." (For clinical trials, collateral data might include age, sex, and limb loss - data identifying a person will not be made public.) Similarly compare $BMI_{metric} = w/h^2$ or $BMI_{mperial} = 703*w/h^2$. Also $BMI_{New_{metric}} = 1.3*w/h^2$.5.

^{aa} By body caliper, or solid wire bent to an Ω -shape, tightened to graze the skin.

bb Scaling variables M, N, & **∆** become constants with more datasets.

cc https://www.cdc.gov/nchs/fastats/body-measurements.htm - cf. Table 1.

^{dd} CCTV seminar for 200 cardiologists by Fayaz A Schall from Washington Adventist Hosp., Alzheimer's therapies, occupational tool adaptations for persons with disabilities, and early use of amniocentesis (aired over 200 PBS stations and winner of Corporation for Public Broadcasting "Outstanding Documentary"), etc.

Editor **Yehia Y Mishriki MD** is a widely published clinician and educator specializing in internal medicine. Now Emeritus Professor of Medicine of the Lehigh Valley Health Network, his career spanned decades with Eastern Pennsylvania's largest healthcare provider. He's cited frequently having authored more than one hundred scholarly works on medical science, diagnosis, and treatment.

Editor **Gilbert H Aykroyd BSEE** has served on several PBS national committees, the Philadelphia Society of Motion Picture & Television Engineers (SMPTE), and the Arizona Society of Broadcast Engineers (SBE). As CTO of #9 market KAET-TV8, he supervised construction of and moving operations to a new 6-story building in downtown Phoenix. He converted TV8 and WLVT-TV39 to digital HD.

Reviewers: **Terrill E Theman MD** & professor of engineering; **James J Boylan MD**; **John S Kintzer MD**

Two upon whose shoulders we stand (quoting other researchers)

"A founder of the social sciences, **Adolphe Quetelet's** (1796~1874) 1835 Treatise of Man and the development of his faculties is hailed as 'one of the greatest books of the 19th century.' A tireless promoter of statistical data collection and analysis, Quetelet organized in 1853 the first International Statistical Congress." ²⁰ In 1972 Quetelet's Index of 1832 became the mathematical basis for Ancel Keys' BMI.

"Centenarian physiologist **Ancel Keys'** (1904~2004) 1972 Seven Countries Study culminated more than 20 years of effort to discredit the accepted measure of obesity, weight-for-height." In it he coined Quetelet's Index as "BMI," universally used for 50 years, and leading to its 1995 categories,. [Yet] "Keys's study failed to demonstrate either BMI or adiposity as superior predictors of heart disease." ²¹

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An abridged version for health professionals has been submitted to medical journals. This original paper contains full math derivations and means of calculating BDI_n.

FilmakerTechnology White Papers are educational, semi-technical on practical topics. They differ from fully scientific ones in references and supplemental math handy in page footnotes and are only informally peer-reviewed. For general readers, they are intended as no less useful. Updates are posted at www.filmaker.com/papers.htm.

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